Mathematics, Mathematical Practice and Communication of Mathematics

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Abstract

Using as example the mathematics of Alan Turing we argue that knowledge - in particular, mathematical knowledge - is locally produced and engaged with the contexts from which it was conceived. This allows us to highlight two issues about mathematics, mathematical practice and the communication of mathematics. The first thing is that asymmetric power relations are strengthened in what is said to be 'technique', in so far as the technique, under an universal and neutral conception, just admit questionings on its own terms. The second thing is that situations of a local conjuncture have a direct influence in the conformation of what is considered 'technique'. Such issues justify the conception of a mathematics that, when rescuing the links with local conditions, makes the understanding easier, configuring thus a more accessible mathematics.

Language of bells

> −Júlia, é anjinho que estão tocando?
> −Não senhora, dona Anica, é pecador.
> −Como assim, Júlia?
> −O camarim do Senhor dos Passos não toca anjinho, só bate defunto...
> −É homem ou mulher, Júlia?
> −É homem, dona Anica; a senhora não vê que é só o grossão?
>
> (Sinos de Goiás, Cora Coralina)

We start these reflections about mathematics with words of Cora Coralina, a Brazilian writer and poet. In "Bells of Goiás" she tells that after a long time away from Goiás, she, Dona Anica, started to misunderstand the language of bells. Seeking an explanation, she appealed to the wisdom of Julia, an old woman, local household worker. From the ballad of the bells, Julia explains each step of the funeral procession. From pathways of procession to the sex of the dead person, everything could be perceived. The understanding of the bells comes from a congruence of thoughts that requires no further explanation for those, as Julia, who are part of the collective. As shared knowledge, these explanations are then omitted. However, as noted Dona Anica, for those who came from the outside, the 'obvious' demand explanations[1]. It is necessary to retrace the links between the language of the bells and the things of life. Precisely in this resides the *wisdom* of Julia: she can *situate* the abstract language in the world, retracing the links between the sounds of the bells and the life of Dona Anica.

Paulo Freire, the Brazilian educator, also took into account these links. In the early sixties, he changed the practice of adult education in Brazil. He replaced the traditional method of repetition of words and reproduction of context-free phrases by a strong commitment with the life experience of each group or individual. His proposal for adult literacy considered words and topics from daily life and phrases contextualized in critical reflections about the social condition. But then, a military coup happened in Brazil in 1964. Under the eyes of the repressive government, Freire's approach sounded subversive and was interrupted. Paulo Freire continued his work in Education in the exile and on the same basis: critical reflections, dialogue, and participation. For some time, he was better known abroad than in Brazil. Now, he is recognized as one of the greatest

1 "Muito tempo longe de Goiás, passei a desentender a linguagem dos bronzes e querendo me inteirar das ocorrências badaladas, apelo para a instância superior representada por Júlia, com seus 50 anos de casa velha e sua sabedoria que vai pelo espaço." (Sinos de Goiás, Cora Coralina)

thinkers in Education, who changed minds and paths in the Brazilian understandings and practices of Education.

From the sounds of the bells, we see that language demands a connection to things of life-world. In the same way, as argued Paulo Freire, literacy has a lot to do with the comprehension of Man in his place of living, his achievements and his relationship with his world. Why then should this be different with mathematics?

Language of God

> In my generation of Brazilians in the Northeast, when we spoke about mathematics, it was something for Gods … or geniuses … a concession was made for genius, guys who could do mathematics without being God.[2]

These are the words of Paulo Freire in 1996, one year before his death, in an interview by Ubiratan d'Ambrosio. As well as Freire, Ubiratan d'Ambrosio, also conducted his investigations with eyes turned to men, their behaviour, life, and doings. The Brazilian mathematician Ubiratan proposed the mathematics of daily life arguing that, from case studies in communities, it is possible to raise evidences that men construct their mathematical abstractions as responses to their needs, thus weakening the bases of an universal, unquestionable and untouchable body of knowledge. The latter configures the mathematics of Gods, completely disconnected from things of life, as the mathematics that was taught to Paulo Freire when he was a boy in the thirties. It is such an abstract construction that its understanding seems to require a special kind of talent, a thing of geniuses.

For many years, the field of Sociology of Knowledge reinforced this view considering mathematics as a kind of thinking that demanded a specific mode of understanding. (Bloor, 1991) Nowadays, as in the thirties, it is not uncommon to admit that

2 (…) "na minha geração de brasileiras e brasileiros lá no Nordeste, quando a gente falava em matemática, era um negócio para deuses ou gênios. Se fazia uma concessão para o sujeito genial que podia fazer matemática sem ser deus."

mathematical entities exist on their own, independently from human thought and life (Bernays, 1935; Chateaubriand, 2012). For a brief example, we go to the Wikipedia, a device that spreads, as much as performs, shared knowledge. We observe how the Brazilian page of Wikipedia explains the term "Natural Sciences" (http://pt.wikipedia.org/wiki/Ci%C3%AAncias_naturais, March,2014):

> The term *Natural Science* is also used to distinguish those fields that use the scientific method to study Nature from the fields of Social Sciences and Humanities, which use the scientific method to study human behaviour and society, and from formal sciences such as Mathematics and Logic that use *a different methodology.*

There is a link on "different methodology" but, up to the moment we write this paper, this goes to a page "under construction", possibly waiting for someone able to explain what kind of difference there is, that makes mathematics and logic such a difficult thing.

Language of the collectives

We come with two explanations in the sense of considering that, like any other kind of knowledge, mathematics is born stepped in worldly things. We start by a sociologist of knowledge, Ludwik Fleck, in the thirties, and then we turn to a philosopher of mathematics, Bertrand Russell, in the early twentieth century.

> There is no emotionless statement as such nor pure rationality as such. How could these states be established? There is only agreement or difference between feelings, and the uniform agreement in the emotions of a society is, in this context, called freedom from emotions. This permits a type of thinking that is formal and schematic, and that can be couched in words and sentences and hence communicated without major deformation. The power of establishing independent existences is conceded to it emotively. Such thinking is called rational. Fleck (1935:49)

For Ludwik Fleck, abstract entities (such as those that appear in mathematical discourse as independent existences) result from a process of purification that comes from an agreement, which, once cleaned of feelings, generates the objective, impartial and universal rational discourse. There is thus a social component that resides in the basis of what is called rational thinking.

From Bertrand Russell we bring an excerpt where he justifies the use of induction under the basis of a collective experience, and *not* as a rational chain of steps. Induction is one of the methods by which the mathematician makes generalizations: he starts from a set where there is a first element, and a notion of successor such that he always knows the next element to be taken. Then, making sure that something is true for the first element, and also making sure that this same thing will always be true for a next element once that it is true for the previous one, the mathematician feels comfortable to extend this to the whole set saying 'for all element of the set, *this* holds'. In the following words note that Russell replaces the certainty of a proof by 'some reason in favour of'.

> But the real question is: Do *any* number of cases of a law being fulfilled in the past afford evidence that it will be fulfilled in the future? If not, it becomes plain that we have no ground whatever for expecting the sun to rise tomorrow, or for expecting the bread we shall eat at our next meal not to poison us, or for any of the other scarcely conscious expectations that control our daily lives. It is to be observed that all such expectations are only *probable*; thus we have not to seek for a proof that they *must* be fulfilled, but only for some reason in favour of the view that they are *likely* to be fulfilled. (Russell,1912)

Essentially, both Fleck and Russell seem to agree that what we usually take as a pure abstract (objective) thought is, in fact, an assemblage in which worldly things are not of minor importance.

A Situated Mathematics

We bring an example where the communication of mathematics reveals the local conditions of its production. It makes it clear that mathematical knowledge is locally produced and engaged in the contexts from which it was conceived. We follow the work of Alan Turing in the search of the formalization of the concept of "mechanical", "computable", or "machine". At the same time as Turing (1936) several other mathematicians were involved with the same problem and made proposals. Although they worked separately, the equivalence between these proposals was proved. In the words of Fleck, there was 'an agreement of feelings' on what they thought to be 'mechanical'. What has not been established however is whether any of these proposals 'in fact' formalized the concept of 'mechanical': the chasm between reality and representation. Faced with the impossibility of completely formal support bases, Hartley Rogers (1967:20), a mathematician, remarks on the need of collective agreement:

> The claim that each of the standard formal characterizations provides satisfactory counterparts to the informal notions of algorithm and algorithmic function cannot be proved. It must be accepted or rejected on grounds that are, in large part, empirical.

While many mathematicians have embraced abstract ways, making use of a very elaborate mathematics to formalize the concept of 'mechanical', Turing decided to observe and describe the action of man when computing with all materialities and human needs involved in this process: paper, pencils, the need to interrupt the calculation and resume later, the limitations of a man to analyze a sequence of symbols at a glance. From this, Turing built an abstract representation: a box with a read head that ran on a tape of symbols. The head could read and write symbols on the tape according to predefined rules. Based on this abstract model, Turing proposed that 'mechanic' would be all that could be computed by this machine. It is amazing the fact that Turing chose the path of observation and recording, since the environment in which he lived extolled the supposedly pure deductive reasoning. Even more surprising is the fact that his proposal immediately convinced the mathematicians, who preferred Turing's proposal to their own.

'What are the possible processes which can be carried out in computing a number?' was the starting question of Turing (1936). He made the (imagined) experiment of following a human actor in the process of calculating and considering the materiality of the assemblage (human + pencil + paper). This approach reenacts the process that a human actant, equipped with pencil and paper, enacts to perform a calculation. As an ethnographer who follows traces and behaviour, Turing took into account the details of the activity of the computer: 'We may now construct a machine to do the work of this computer' (Turing, 1936). The word 'computer' was the term adopted by him to describe the assemblage (human+ pencil + paper) in the act of calculation – at that time the computer (the machine) had not yet been constructed. The 1936 paper shows how Turing held obsessively close to materiality as he observed and traced each step in the process of a calculation. He literally stated that the abstract machine he had conceived possessed all the materiality that corresponded to the materiality of the calculating activity of a person with a pencil and paper: 'We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions' (Turing, 1936). This correspondence is accurate enough to consider situations where the man takes a break, so interrupting calculations to resume them later on: 'It is always possible for the computer to break off from his work, to go away and forget all about it, and later to come back and go on with it. If he does this, he must leave a note of instructions (written in some standard form) explaining how the work is to be continued. This note is the counterpart of the "state of mind". (Turing,1936).

This process resulted in an ‘abstract’ conception (apparently devoid of materiality) which is, nevertheless, clearly embodied in the computer machines which emerged shortly afterwards. Although Turing never mentioned this anthropological technique, his approach to the issues of mathematics, and particularly the question of what is ‘calculable’ or ‘computable’ is precisely ethnographic. Turing’s commitment to the notion of materiality is also visible in the 1950 paper “Computing Machinery and Intelligence”, where he takes into account new elements for a new time: ‘He [the man doing calculation] has also an unlimited supply of paper on which he does his calculations. He may also do his multiplications and additions on a ‘desk machine’’. For Turing, more than just a style of writing, adherence to empirical facts, inductive reasoning, and local conjunctures were a way of thinking.

‘This is in accordance with experience’ wrote Turing (1936) when concluding that the machine should take into account one symbol at a time, since a human would not be able to decide at a glance if two sequences 9999999999999999 and 999999999999999 are the same. It is clearly evident here that Turing seems to have realized that mathematics and a kind of immediate experience overlap only in a limited way: the inequality between 99 and 999, for example, would be ‘immediately’ (i.e. without mediation) perceived.

By adopting an empirical attitude, Turing faced the problem of formalizing an intuitive notion:

> The arguments which I shall use are of three kinds. (a) A direct appeal to intuition. (b) A proof of the equivalence of two definitions (in case the new definition has a greater intuitive appeal). (c) Giving examples of large classes of numbers which are computable. (Turing, 1936).

What emerged in Turing’s approach was the empiricist nature of knowledge construction, which he reached through ethnographic research. Hence, a study of Turing’s way of working draws attention to approaches that consider diverse factors of diverse natures, these being on the same scale or order as what is usually indicated as 'objective factors' in the construction of the 'objective' facts of Science. These approaches may shed new light on questions about the neutrality and universality of mathematical knowledge.

Language of Authority

> Authoritarianism is not necessarily associated with physical repression. It also happens in actions that are supported by the 'authority argument'. 'This is so, the technique has already said. Don't question, just apply'[3] (Freire,1983)

Authority and oppression were constant concerns of Paulo Freire as conditions incompatible with Education. For him, an educational action starts when man seeks to become aware about his social condition. This is a situated approach: understanding a man in his place, at his time, with his doings. In the same way, a technique embodies its whole historical process, thus bringing its links to its place of enunciation. We see, however, that scientific descriptions seem to be free of such links. The more distanced, neutral and universal seems to be the description, the more 'scientific' seems to be the subject. Again, we resort to the Wikipedia page for 'Natural Sciences', but at this time, we go to the English entry (http://en.wikipedia.org/wiki/Natural_science), to see how mathematics is presented as a different kind of knowledge:

> The term "natural science" is used to distinguish the subject from the social sciences, such as economics, psychology and sociology, which apply the scientific method to the study of human behavior and social patterns; the humanities, which use a critical or analytical approach to study the human condition; and the formal sciences such as mathematics and logic, which use an a priori, as opposed to empirical methodology to study formal systems.

There is a link in "a priori" (http://en.wikipedia.org/wiki/A_priori_and_a_posteriori):

> *A priori* knowledge or justification is independent of experience (for example "All bachelors are unmarried"). Galen Strawson has stated that an *a*

3 "O autoritarismo não está necessariamente associado a repressões físicas. Dá-se também nas ações que se fundamentam no "argumento da autoridade". "Isto é assim porque é – a técnica já o disse – não há o que discordar, mas sim, que aplicar."

priori argument is one in which "you can see that it is true just lying on your couch".

As we see, mathematics is commonly presented as a-historical, as if mathematical objects were ready in the world, waiting for being discovered by mathematicians. Hiding the construction process of thought, this practice precludes possibilities of questionings. It is an authoritarian practice that reinforces a power configuration around mathematics. In fact, for common sense, numbers 'depersonalize', make the analysis (and reality) less subjective, less dependent on local circumstances, free of tacit knowledge, and so, when mathematicized, propositions and arguments seem more reliable. A mathematical proof, even if not understood, often has the power of persuasion: 'It is proved then it is right!' As the language of the bells, a mathematics detached from reality will always need someone like Julia, wisely enough to redo the links between the abstract language and life-world.

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